AMERICAN MATHEMATICAL SOCIETY MathSciNet Mathematical Reviews on the Web Previous Up Next Article

MR913535 (89d:01007) 01A20 (00A69 01A35 01A40 92A27) Neumaier, Wilfried (D-TBNG)

★ Was ist ein Tonsystem? (German) [What is a tone system?]

Eine historisch-systematische Theorie der abendländischen Tonsysteme, gegründet auf die antiken Theoretiker Aristoxenos, Eukleides und Ptolemaios, dargestellt mit Mitteln der modernen Algebra.

A historical-systematic theory of occidental tone systems, based on the ancient theoreticians Aristoxenos, Euclid and Ptolemy, presented with the tools of modern algebra].

Quellen und Studien zur Musikgeschichte von der Antike bis in die Gegenwart [Sources and Studies on the History of Music from Antiquity to the Present], 9.

Verlag Peter D. Lang, Frankfurt am Main, 1986. 261 pp. sFr. 57.00. ISBN 3-8204-9492-8

The aim of this book is to gain new insights into the nature of tonal systems through an integration of historical schemes into an axiomatic system, formulated in a language inspired by contemporary abstract algebra but organized according to the principle that concepts, definitions and axioms should belong specifically to the domain of auditive experience.

Part A of the book thus builds up the concepts necessary to construct an "auditive tonal system" (as differentiated from "acoustical systems" based on frequency, monochord, etc., and from mathematical models). Part B makes a new start, describing two paradigmatic approaches from Greek Antiquity, the "auditive" system of Aristoxenos and the Pythagorean tradition based on ratios as (re)formulated in Euclid's *Sectio canonis*. Both systems are analyzed, and their incompatibility is clearly expounded. In the same connection it is shown that while Aristoxenos' axioms are fulfilled in an equal temperament, the idea of temperament is irrelevant to the understanding of Aristoxenos.

A spin-off from the main analysis is a reconstruction of book III of Aristoxenos' *Harmonic elements*, based on strong and clear arguments for what must be original and what must be secondary insertions. A similar analysis of the *Sectio canonis* shows that its second part must be understood as a nonpolemic response to Aristoxenos, which fits a Euclidean authorship but nothing significantly earlier. The logical structures of both works are drawn up in schematic form.

A third ancient writer dealt with here is Ptolemy (though only in his relation to the previous two authors). He is shown to know Aristoxenos only through second-rate Aristoxeneans, and his synthesis is shown to be a theoretically shaky compromise, as it had to be on the mathematical foundation available in Antiquity. Finally, brief references to Boethius are given.

Part C is then an attempt at synthesis on contemporary algebraic foundations. It continues the historical approach, following the development of the Boethian system from Odo of Cluny and Guido of Arezzo until the development of the mean-tone and equal temperament; another strand is the development of a framework into which all historical positions can be fitted, followed by sketches of the actual fitting.

The book is provided with indices of mathematical concepts, of musical concepts, and of names, and with an explanatory list of mathematical concepts (ranging from those of mathematical logic

to those of ordered groups). The list contains a few errors (worst is a corrupt description of the "inverse function"), but nothing which will impede understanding. In a scheme on p. 159 taken from Boethius, "2196" should be changed everywhere to "2916" (three times).

The description of ancient authors gives valuable new insights. Part B, however, is not written as, and should not be used as, a general introduction to ancient harmonic theory.

Reviewed by Jens Høyrup

© Copyright American Mathematical Society 1989, 2007